

HW2 Hints & Partial Solns.

★ Let's form the reduction formula first.

#7. $I_n = \int \sin^n x dx$ (Formula 19 Equation (4))

$$I_n = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2}$$

Now $A_n = \int_0^\pi \sin^n x dx$ ← definite integral

$$\Rightarrow A_n = -\frac{1}{n} \sin^{n-1} x \cos x \Big|_{x=0}^{x=\pi} + \frac{n-1}{n} A_{n-2}$$

(because $\sin \pi = \sin 0 = 0$, the 2nd term is 0)

★ $A_n = \frac{n-1}{n} A_{n-2}$ ← simplified !! 😊

Now attack the problems:

(a) $A_0 = \int_0^\pi \sin^0 x dx = \int_0^\pi 1 dx = \pi$

$$A_1 = \int_0^\pi \sin x dx = -\cos x \Big|_{x=0}^{x=\pi} = -(-1-1) = 2$$

(b) Use Formula (★),

$$A_5 = \frac{4}{5} A_3 = \frac{4}{5} \left(\frac{2}{3} A_1 \right) = \frac{8}{15} \cdot 2 = \frac{16}{15}$$

$$A_6 = \frac{5}{6} A_4 = \frac{5}{6} \left(\frac{3}{4} A_2 \right) = \frac{5}{6} \cdot \frac{3}{4} \left(\frac{1}{2} A_0 \right) = \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \pi = \frac{5}{16} \pi$$

$$A_7 = \frac{6}{7} A_5 = \frac{6}{7} \cdot \frac{16}{15}$$

(c) Hint:

$$\sin x \leq 1$$

$$\Rightarrow \sin x \cdot \sin^{n-1} x \leq \sin^{n-1} x$$

(when $0 \leq x \leq \pi$)

(d) Problem isn't clear.

No need to do this part.

#8. (Hint = separate one "cos x" out !! As what we did for $\int \cos^3 x dx$).

$$\int \cos^n x dx = \int \cos^{n-1} x \cos x dx$$

Now its time for IBP.

Also, $\sin^2 x = 1 - \cos^2 x$ could be helpful.

#12. IBP \rightarrow IBP.

#15 Use Formula (the 1st Equation of Page 20).

#18.
$$? = \int \frac{1}{\underbrace{(1+x^2)^2}} + \frac{x}{\underbrace{(1+x^2)^2}} dx$$

Think about it?

There's one technique that could solve this integral directly.

P26. #8. (Hint: Do we really need Partial Fraction for this Problem?)
Isn't there an easier way out?

#16 (Hint = try a u-sub and then Partial Fractions)

20.
$$\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{1}{x-1}$$

repeated linear.

↓
It's a "Arctan" case.
~~look at~~ Problem #7 in our worksheet
may help!